SAT/ACT MATH GUIDE

SAT TEST-TAKERS SHOULD BECOME FAMILIAR WITH THE MATH FORMULAS LOCATED ON THE FRONT OF SECTIONS 3/4 SAT MATH.

SAT TEST-TAKERS MUST ALSO BECOME FAMILIAR WITH A NUMBER OF OTHER SAT MATH FORMULAS EMPLOYED ON THE TEST.

ACT TEST-TAKERS SHOULD BECOME FAMILIAR WITH ALL RELEVANT ACT MATH FORMULAS CONTAINED IN THIS COMPREHENSIVE GUIDE.

[PLEASE NOTE: SOMETIMES SAT/ACT MATH TESTS WILL PROVIDE MATH FORMULAS FOR UNUSUAL MATH QUESTIONS.]

SAT/ACT STRATEGIES

WHERE THE QUESTION INCLUDES VARIABLES WITH NO SPECIFIC ANSWER, ONE COMMON TACTIC IS TO SUBSTITUTE YOUR OWN NUMBERS FOR THE VARIABLES, ANSWER THE QUESTION, THEN SUBSTITUTE THOSE SAME NUMBERS INTO THE VARIABLES CONTAINED IN THE ANSWER CHOICES LOOKING FOR THE SAME ANSWER.

FOR EXAMPLE, IN ORDER TO ACCOMMODATE x HIKERS, THE y GROUP OF SHERPAS NEEDED TO CARRY z POUNDS OF EQUIPMENT PER HIKER. ASSUMING EACH HIKER REQUIRES AN EQUAL AMOUNT OF HIKING EQUIPMENT AND EACH SHERPA CARRIES AN EQUAL AMOUNT OF HIKING EQUIPMENT, WHICH FORMULA CORRECTLY CALCULATES (z)(y) IN TERMS OF x WHERE THERE ARE FOUR LESS HIKERS.

MAKE x EQUAL 12, y EQUAL 4 AND z EQUAL 2; THEREFORE, EACH OF THE 4 SHERPAS MUST CARRY 6 POUNDS OF EQUIPMENT EACH FOR A TOTAL OF 24 POUNDS. WHERE x EQUALS 8, y EQUALS 4 AND z EQUALS 2, EACH OF THE 4 SHERPAS MUST CARRY 4 POUNDS OF EQUIPMENT EACH FOR A TOTAL OF 16 POUNDS.

WHERE THE QUESTION IS AMBIGUOUS, THE TEST-TAKER SHOULD EMPLOY THE P.I.A. (PLUG IN ANSWER CHOICES) METHOD OF ANSWERING THE QUESTION.

THE SAT/ACT TEST MAKERS DISDAIN RIGIDITY; REWARD FLEXIBILITY. PART OF DOING WELL IS TO FIGURE OUT THE

BEST METHOD FOR ANSWERING EACH QUESTION—DIRECTLY TACKLING THE QUESTION OR INDIRECTLY TACKLING THE QUESTION BY EMPLOYING P.I.N. OR P.I.A.

THERE ARE 58 QUESTIONS ON THE MATH PORTION OF THE SAT; 60 QUESTIONS ON THE MATH PORTION OF THE ACT. THE P.I.N. METHOD AND THE P.I.A. METHOD SHOULD BE EMPLOYED TO ANSWER APPROXIMATELY FIFTEEN TO TWENTY PERCENT OF THE QUESTIONS ON BOTH THE MATH PORTIONS OF THE SAT AND THE ACT.

P.I.N. METHOD

BELLUS HAD A COLLECTION OF x PET ROCKS. AFTER BELLUS ACQUIRED MORE PET ROCKS AT THE ANNUAL PET ROCK COLLECTORS' CONVENTION, HER COLLECTION INCREASED BY 30 PERCENT, BUT, AFTER

SELLING SOME OF HER CHERISHED PET ROCKS, HER COLLECTION DECREASED BY 40 PERCENT. WHAT PERCENT OF THE ORIGINAL PET ROCK COLLECTION DOES BELLUS HAVE LEFT?

START BY MAKING x EQUAL TO 100 [WITH PERCENT QUESTIONS, ALWAYS USE 10 OR 100.]. [WHEN WORKING WITH PERCENTS YOU CAN EMPLOY A ONE OR TWO STEP METHOD: ONE STEP METHOD: WHEN YOU INCREASE, SIMPLY ADD THE INCREASED AMOUNT — HERE 30% —TO 100 AND MULTIPLY. 130 % IS 1.3 AS A DECIMAL [FOR PERCENT, CONVERT A PERCENT INTO A DECIMAL BY MOVING THE DECIMAL POINT TWO PLACES TO THE LEFT. TO CONVERT A PERCENT INTO A FRACTION SIMPLY PLACE THE PERCENT OVER 100: 130/100]. TWO STEP METHOD: TAKE INCREASED PERCENT OF VALUE HERE, 30% AND MULTIPLY: (100)(.3) OR (100)(30/100). EITHER WAY, YOUR ANSWER IS THE SAME: 130.

WHEN YOU DECREASE, USING THE ONE STEP METHOD: SIMPLY SUBTRACT THE DECREASED AMOUNT FROM 100 — HERE, 40 MINUS 100 EQUALS 60. NOW MULTIPLY THE NEW AMOUNT—130—TIMES 60 PERCENT: (130)(.6 OR 60/100), OR TWO STEP METHOD: TAKE 40% OF 130 — THEN SUBTRACT IT

FROM 130. EITHER METHOD WILL RESULT IN THE SAME ANSWER: (130)(.6) EQUALS 78.

FINALLY, FORMULA FOR PERCENT CHANGE: INCREASE (OR DECREASE) PERCENT CHANGE IS THE SAME FORMULA: (CHANGE IN AMOUNT)/(ORIGINAL AMOUNT) (100 - 78)/100 EQUALS 22/100 MEANING A 22% DECREASE. [BE CAREFUL IF QUESTION STATES JANUS HAS $10,500 IN BLACK ROCK STOCK WHICH INCREASED BY 25% IN THE FALL, AND DECREASED BY 45% IN THE SPRING, YOU CAN NEVER PUT THESE TWO VALUES TOGETHER FOR A TOTAL DECREASE OF 20%. INSTEAD, YOU MUST FIRST INCREASE THE $10,500 BY 25%: MULTIPLY ($10,500)(1.25) = $13,125, THEN DECREASE $13,125 BY 45% EQUALS ($13,125)(.55) = $7,218.75. IF YOU HAD COMBINED THE INCREASE OF 25% WITH THE DECREASE OF 45% CALCULATING A COMBINED TOTAL OF A DECREASE OF 20%, YOUR ANSWER WOULD BE ($10,500)(.8) = $8,400, WHICH IS INCORRECT.

ANOTHER SAMPLE P.I.N. QUESTION: PATS WAS HAVING A PARTY FOR A GROUP OF x FRIENDS. IN ANTICIPATION OF THE PARTY, PATS MADE x -3 LOAVES OF BREAD. BEFORE THE PARTY, PATS DISCOVERED THAT TWELVE ADDITIONAL FRIENDS WOULD BE COMING TO HER SOIRÉE. IN TERMS OF x HOW MANY ADDITIONAL LOAVES OF BREAD SHOULD PATS MAKE

SO THAT HER NEW GROUP OF GUESTS WOULD RECEIVE THE SAME AMOUNT OF BREAD?

USING P.I.N., LET x EQUAL 9 [USE EASY NUMBERS—HERE A MULTIPLE OF 3.]. THEREFORE PATS MADE 6 LOAVES OF BREAD FOR HER ORIGINAL LIST OF NINE FRIENDS, WITH EACH FRIEND RECEIVING 6/9 OR 2/3 LOAVES OF BREAD.

[ALWAYS WRITE DOWN ALL YOUR NUMBERS HERE, USING YOUR NUMBERS SOLVE/WRITE DOWN ANSWER TO QUESTION ASKED.]

BUT, BEFORE THE PARTY, PATS DISCOVERED SHE WOULD NOW BE HOSTING 9 + 12 = 21 FRIENDS. ANSWERING THE QUESTION ASKED: HOW MANY ADDITIONAL LOAVES OF BREAD DOES PATS NEED TO MAKE. FIRST, CALCULATE HOW MANY LOAVES OF BREAD PATS NEEDS TO MAKE: (21)(2/3) WHICH EQUALS 14 LOAVES OF BREAD, BUT PLACING 14 BACK INTO THE QUESTION, SINCE x IS 9, 14 - 9 = 5.

P.I.N. CAN ALSO BE USED TO SOLVE OPEN-ENDED GEOMETRY QUESTIONS:

CIRCLE A HAS A DIAMETER FIVE TIMES AS GREAT AS THE DIAMETER OF CIRCLE B. HOW MANY TIMES

GREATER IS THE AREA OF CIRCLE A TO THE AREA OF CIRCLE B?

SINCE QUESTION SAYS FIVE TIMES AND WE NEED 'EASY' QUANTITIES THAT ARE BOTH A MULTIPLE OF FIVE AND DIVISIBLE BY 2, WE USE 6 FOR THE DIAMETER OF CIRCLE B AND 30 FOR THE DIAMETER OF CIRCLE A, REMEMBERING THAT THE FORMULA FOR CALCULATING THE AREA OF A CIRCLE IS: (pi)(r sq'd) WE THEREFORE NEED THE RADIUS. RECALLING THAT THE RADIUS IS HALF OF THE DIAMETER, WE KNOW THE RADIUS OF CIRCLE B IS 3 AND THE AREA OF CIRCLE B IS (3 sq'd)(pi) OR 9 pi. SINCE THE DIAMETER OF CIRCLE A IS 30, THE RADIUS IS 15; THEREFORE, THE AREA OF CIRCLE A IS (15 sq'd)(pi) OR 225 pi.

THEREFORE, THE AREA OF CIRCLE A IS 25 TIMES THE AREA OF CIRCLE B.

YOU TRY:

BETS PURCHASED x TICKETS TO A CONCERT. AFTER GIVING A THIRD OF THE TICKETS TO LESS THEN REPURCHASING BACK A FOURTH OF THE TICKETS FROM SAL, WHAT FRACTION OF THE x TICKETS DOES BETS CURRENTLY HAVE?

P.I.A. (PLUG IN ANSWER CHOICES)

IN THE FOLLOWING PARABOLA, f(a) = a sq'd - a - 6. IF f(0) = -6, WHAT VALUE OF a EQUALS 0?

PLUG IN ANSWER CHOICES:

A) 1 (1 sq'd) - 1 - 6 = -6
B) 2 (2 sq'd) - 2 - 6 = -4
C) -1 (-1 sq'd) - (-1) - 6 = -4
D) -2 (-2 sq'd) - (-2) - 6 = 0

CORRECT ANSWER IS D).

YOU TRY:

IF 3 TIMES A NUMBER EQUALS THE NUMBER MINUS FOUR, WHAT IS THE NUMBER?

A) -1
B) 0
C) -2
D) 1

P.I.N./P.I.A. COMBINATION

A QUANTITY IS INCREASED BY 35% OF ITS VALUE. THE RESULTING VALUE IS x. WHICH EXPRESSION GIVES THE VALUE OF THE ORIGINAL QUANTITY IN TERMS OF x?

RECALL: WHEN DEALING WITH PERCENTS, USE 100. INCREASING 100 BY 35% IS 135. NOW, SOLVE FOR ORIGINAL VALUE OF 100, PLUGGING IN 135 FOR x.

A) x/1.65   135/1.65 = 81.8181…
B) x/0.65   135/0.65 = 207.692308
C) x/1.35   135/1.35 = 100
D) x/0.35   135/0.35 = 385.714286

CORRECT ANSWER IS C).

YOU TRY:

WHICH OF THE FOLLOWING FUNCTIONS HAS THE LOWEST VALUE OF t, WHERE t IS A NEGATIVE CONSTANT?

A) f(t) = tx sq'd
B) f(t) = -(x + t) sq'd
C) f(t) = -tx sq'd
D) f(t) = -x sq'd + t

GEOMETRY

CIRCLES

P.I.N. (PLUG IN A NUMBER)

A COMMON TRICK ON SAT/ACT IS TO PROVIDE THE DIAMETER OF THE CIRCLE IN THE QUESTION KNOWING THAT IN ORDER TO SOLVE THE QUESTION, THE TEST TAKER NEEDS THE RADIUS. FOR EXAMPLE, IF CIRCLE A HAS A DIAMETER WHICH IS THREE TIMES THE DIAMETER OF CIRCLE B, HOW MUCH GREATER IS THE AREA OF CIRCLE A TO THE AREA OF CIRCLE B. BEST WAY TO SOLVE IS TO P.I.N. (PLUG IN A NUMBER). [WHENEVER YOU HAVE A NON-SPECIFIC QUESTION, YOU CAN ALWAYS PLUG IN A NUMBER AND SOLVE THE QUESTION THEN LOCATE AMSWER. {HOWEVER, TRY TO AVOID USING 1 0R 2 WHEN PLUGGING IN A NUMBER; INSTEAD START WITH 3.}]. DIAMETER OF CIRCLE B IS 6, MAKING DIAMETER OF CIRCLE A 18, WHICH MEANS THE RADIUS OF CIRCLE B IS 3 WHICH MEANS THE AREA OF CIRCLE B IS 9 PI. THE DIAMETER OF CIRCLE A IS 18 WHICH MEANS THE RADIUS OF CIRCLE A IS 9 WHICH MEANS THE AREA OF CIRCLE A IS 81 PI. THEREFORE, THE AREA OF CIRCLE A IS NINE TIMES THE AREA OF CIRCLE B.

FIGURES CONTAINED WITHIN A CIRCLE

OBTAINING THE ANGLE MEASURES OF A FIGURE CONTAINED WITHIN A CIRCLE REQUIRES YOU TO RECALL THAT THE ANGLES OPPOSITE THE SIDES OF A TRIANGLE WITH THE SAME LENGTH ARE EQUAL IN MEASURE. THUS THE ANGLES OPPOSITE THE RADII AND THE DIAMETERS OF THE CIRCLE WHICH FORM THE SIDES OF A TRIANGLE INSIDE THE CIRCLE ARE THE SAME MEASURE.

INTERPLAY BETWEEN CENTRAL ANGLE AND ANGLE TOUCHING CIRCUMFERENCE OF CIRCLE

WHENEVER A PARTIAL TRIANGLE IS FORMED ORIGINATING FROM THE CIRCUMFERENCE OF THE CIRCLE WHICH THEN COMES BACK IN ON ITSELF TOUCHING THE CENTER OF THE CIRCLE, THE ANGLE OF THIS FIGURE WHICH TOUCHES THE CIRCUMFERENCE OF THE CIRCLE IS HALF THE MEASURE OF THE CENTRAL ANGLE OF THE CIRCLE.

EQUATION OF A CIRCLE

(x-h) sq'd + (y-k) sq'd = r sq'd WHERE (h,k) ARE THE COORDINATE POINTS OF THE CENTER OF THE CIRCLE AND (x,y) ARE ALL THE COORDINATE POINTS AROUND THE CIRCUMFERENCE OF THE CIRCLE IN THE COORDINATE PLANE.

[PLEASE NOTE THESE TYPES OF QUESTIONS ALWAYS UTILIZE THE CIRCLE IN THE GEOMETRIC PLANE. PLEASE NOTE FURTHER HOW THE MINUS/PLUS SIGN IN THE EQUATION OF A CIRCLE NEGATES THE CENTER: FOR EXAMPLE, (x-9) sq'd + (y+1) sq'd HAS CENTER (9,-1).

[FYI: IN ALL MATHEMATICAL EQUATIONS, THERE IS A MINUS SIGN INSIDE THE PARENTHESES AND A PLUS SIGN OUTSIDE THE PARENTHESES.]

EXAMPLE QUESTION: x sq'd - 10x + y sq'd + 16y = -8
WHAT IS THE DIAMETER OF THE CIRCLE?

STEP ONE: DIVIDE THE 10 FROM 10x BY 2 GIVING YOU 5 WHICH YOU THEN SQUARE PRODUCING 25 WHICH YOU THEN ADD BACK TO BOTH SIDES OF THE EQUATION.

[RECALL: WHATEVER YOU DO ON ONE SIDE OF THE EQUATION, YOU MUST DO ON THE OTHER SIDE OF THE EQUATION.]

THEN DIVIDE THE 16 FROM 16y BY 2 GIVING YOU 8 WHICH YOU THEN SQUARE PRODUCING 64 WHICH YOU THEN ADD BACK TO BOTH SIDES OF THE EQUATION RESULTING IN THE ADDITION OF 25 AND 64 TO -8 PRODUCING 81 WHICH REPRESENTS r sq'd, TAKING THE SQUARE ROOT OF 81 PRODUCES 9 WHICH IS THE RADIUS OF THE CIRCLE, BUT SINCE WE NEED THE DIAMETER OF THE CIRCLE AND SINCE THE DIAMETER IS TWICE THE RADIUS, THE CORRECT ANSWER IS 18.

[PLEASE NOTE: ON THE MORE DIFFICULT SAT/ACT MATH QUESTIONS THE TEST-MAKER WILL DELIBERATELY FORCE THE TEST-TAKER TO FIRST MAKE VARIOUS COMPLICATED COMPUTATIONS, SOLVING FOR AN ANSWER WHICH THE TEST-TAKER WILL THEN NEED TO FURTHER MANIPULATE IN ORDER TO ANSWER THE ULTIMATE QUESTION. IN THE SAMPLE QUESTION ABOVE, FIRST WE HAD TO MANIPULATE THE QUESTION IN ORDER TO SOLVE FOR THE RADIUS; HOWEVER, THE ULTIMATE QUESTION ASKS FOR THE DIAMETER. WORSE STILL, ON THIS QUESTION, WITH MULTIPLE CHOICE QUESTIONS ON THE HARDER SAT MATH QUESTIONS (TYPICALLY, QUESTIONS 10 -15 OF SECTION 3 NO CALCULATOR

MATH AND QUESTIONS 1-30 OF SECTION 4 CALCULATOR MATH AS WELL AS ALL 60 QUESTIONS FROM THE ACT MATH TEST. QUESTIONS 16-20 FROM SECTION 3 SAT MATH AND QUESTIONS 31-38 FROM SECTION 4 SAT MATH ARE STUDENT-GENERATED QUESTIONS.) THE RADIUS WILL BE LISTED AS A POSSIBLE (INCORRECT) ANSWER CHOICE IN ORDER TO DECEIVE THE TEST TAKER. IN MY EQUATION OF A CIRCLE QUESTION ABOVE, I DELIBERATELY PROVIDED YOU WITH THE DIAMETER OF THE CIRCLE, KNOWING THAT IN ORDER TO ANSWER MY QUESTION, YOU FIRST NEEDED TO CONVERT THE DIAMETER OF THE CIRCLE INTO THE RADIUS OF THE CIRCLE.]

RE-WRITING THE EQUATION BACK INTO THE EQUATION OF A CIRCLE PRODUCES:

x sq’d - 10x + 25 + y sq’d + 16y + 64 = 81

(x - 5) sq’d + (y + 8) sq’d = 81

THEREFORE, WE KNOW THAT (5,-8) IS THE CENTER OF THE CIRCLE WITH RADIUS 9; HOWEVER, BEING A CONSCIENTIOUS TEST-TAKER, WE RECALL THAT THE QUESTION ASKS FOR THE DIAMETER, NOT THE RADIUS. AND SINCE DIAMETER EQUALS TWICE THE RADIUS, THE CORRECT ANSWER IS 18.

YOU TRY:

WHAT IS THE CENTER AND RADIUS OF THE CIRCLE WITH THE FOLLOWING EQUATION:

x sq’d + 36 + y sq’d - 144 = -7

WHAT IS THE y COORDINATE OF A CIRCLE WITH CENTER (-4,5), x COORDINATE OF 2 AND A DIAMETER OF 20?

DISTANCE FORMULA

Sq Root (x(1) - x(2)) sq’d + (y(1) - y(2)) sq’d

THIS FORMULA IS USED TO ASCERTAIN THE DISTANCE BETWEEN TWO COORDINATE POINTS IN THE x,y PLANE

EXAMPLE QUESTION: WHAT IS THE DISTANCE BETWEEN (7,-8) AND (11,-2)?

Sq Root (7 - 11) sq’d + (-8 + (-2)) sq’d EQUALS Sq Root (-4) sq’d + (-10) sq’d WHICH EQUALS Sq root (16) + (100) WHICH EQUALS sq root (116) [IN ORDER TO SIMPLIFY Sq Root OF ANY VALUE TRY AND FIND THE LARGEST

PERFECT SQUARE THAT—HERE—DIVIDES INTO 116: Sq Root (4)(19) SINCE THE SQUARE ROOT OF 4 IS 2 THE ANSWER IS 2 Sq Root (19)

MIDPOINT FORMULA

(x(1) + x(2) / 2 , y(1) + y(2) / 2)

MIDPOINT IS THE COORDINATE POINT THAT IS EQUIDISTANT FROM THE TWO ENDPOINTS.

FOR EXAMPLE, WHAT IS THE MIDPOINT BETWEEN (-4,7) AND (5, -3)?

PLUGGING BACK INTO THE MIDPOINT FORMULA:
(-4 + 5 / 2, 7 - (-3) / 2) (1/2, 5 (10/2)) ANSWER (1/2, 5).

IF THE MIDPOINT IS (-3, 2) AND ONE ENDPOINT IS (7, 0), WHAT IS THE OTHER ENDPOINT?

(7 + x) / 2 = -3 / 1 AND 0 + y / 2 = 2 / 1

CROSS MULTIPLY: (-3)(2) = (1)(7+x) AND (2)(2) = (y)(1)
THEREFORE: -6 = 7+x x = -13 AND y = 4 ANS (-13,4)

[CROSS-MULTIPLY EVERY TIME YOU HAVE FRACTIONS FACING EACH OTHER ACROSS AN EQUALS SIGN— YOU CAN ALWAYS TURN AN INTEGER LIKE 6 INTO A FRACTION BY PLACING A 1 IN THE DENOMINATOR THEN CROSS-MULTIPLY.}

EQUATION OF A LINE

$y = mx + b$ WHERE m IS THE SLOPE OF THE LINE, AND b IS THE y INTERCEPT OF THE LINE. (THE y INTERCEPT IS WHERE THE x COORDINATE IS ZERO, AND YOU ARE LOOKING FOR THE MATCHING y COORDINATE. THE x INTERCEPT IS WHERE THE y COORDINATE IS ZERO, AND YOU ARE LOOKING FOR THE MATCHING x COORDINATE.). [ONCE YOU ASCERTAIN THE SLOPE AND THE y INTERCEPT OF THE EQUATION OF A LINE, ALL OTHER x,y COORDINATE POINTS WILL FIT INTO THE LINE.]

SLOPE FORMULA

y(2) - y(1) / x(2) - x(1)

GRAB TWO COORDINATE POINTS FROM THE LINE AND PLUG THEM INTO THE SLOPE FORMULA. [PLEASE NOTE: IT DOES NOT MATTER WHICH COORDINATE POINT YOU LABEL y(2) OR y(1) THE SLOPE WILL BE THE SAME; SO LONG AS YOUR x(2) AND x(1) MATCH. FOR EXAMPLE, (-3,2) AND (1,-7) YOU CAN EITHER SAY -7 - 2 / 1 - (-3) = -(9/4) OR 2 - (-7) / -3 - 1 = -(9/4) WHAT YOU CANNOT SAY IS -7 -2 / -3 -1 = -9/-4 = 9/4 WHICH IS WHY WE MUST ALIGN UP y(2) WITH y(1) AND x(2) WITH x(1) OR VICE VERSA. FINALLY, WHEN YOU SUBTRACT A NEGATIVE, PLACE THE NEGATIVE INSIDE PARENTHESES TO ENSURE YOU CONVERT THE NEGATIVE INTO A POSITIVE.].

WHENEVER THE QUESTION GIVES AN EQUATION OF THE LINE THAT IS NOT IN THE y = mx + b FORMAT, YOU MUST MANIPULATE THAT EQUATION INTO THE PROPER y = mx + b FORMAT BEFORE YOU CAN SOLVE THE PROBLEM.

PARALLEL LINES HAVE THE SAME SLOPE, BUT DIFFERENT y INTERCEPTS WHEREAS PERPENDICULAR LINES HAVE COMPLETELY OPPOSITE SLOPES—THE NEGATIVE RECIPROCAL OF

EACH OTHER—BUT COULD HAVE THE SAME y INTERCEPTS.

FOR EXAMPLE, LINE f HAS COORDINATE POINTS f(-3) = -7 AND f(5) = -3. WHAT IS THE y INTERCEPT OF LINE g (NOT SHOWN) WHICH IS PERPENDICULAR TO LINE f WHERE g(-2) = 1? [BEFORE WE BEGIN TO SOLVE THIS PROBLEM, WE NEED TO RECALL THAT PARALLEL LINES HAVE THE SAME SLOPE/DIFFERENT y INTERCEPTS AND THAT IN THE LANGUAGE OF COORDINATE GEOMETRY g(5) = -7 MEANS LINE g COORDINATE POINT (5,-7) BECAUSE IN THE WORLD OF COORDINATE GEOMETRY WHATEVER NUMBER IS IN THE PARENTHESES IS ALWAYS THE x COORDINATE, AND WHATEVER THE FUNCTION (FORMULA) EQUALS IS ALWAYS THE y COORDINATE. IN OTHER WORDS, COORDINATE GEOMETRY IS SIMPLY A DIFFERENT WAY OF PRESENTING THE x,y COORDINATE POINTS SUCH THAT f(-2) = 7 REPRESENTS COORDINATE POINTS (-2,7).

IN ORDER TO ANSWER THIS QUESTION, FIRST GRAB TWO COORDINATE POINTS FROM LINE g: (-3,-7) AND (5,-3). NOW APPLY THE SLOPE FORMULA: -3 - (-7) / 5 - (-3) WHICH EQUALS -3 + 7 / 5 + 3 [RECALL DOUBLE NEGATIVE TURNS A NEGATIVE INTO A POSITIVE WHICH IS WHY YOU SHOULD ALWAYS PLACE PARENTHESES AROUND NEGATIVE NUMBERS: ESPECIALLY WHEN YOU ARE SUBTRACTING A

NEGATIVE NUMBER. HERE: - (-7) AND - (-3) CONVERTING THE DOUBLE NEGATIVES INTO POSITIVE NUMBERS.] THIS THEREFORE EQUALS -3 + 7 / 5 + 3 WHICH EQUALS 4 / 8 WHICH REDUCES TO 1 / 2 [RECALL WHEN REDUCING A FRACTION INTO ITS LOWEST COMMON DENOMINATOR YOU MUST FIND THE LARGEST NUMBER WHICH WILL DIVIDE INTO BOTH THE NUMERATOR AND THE DENOMINATOR.] [FURTHERMORE, ON THE STUDENT-GENERATED SECTION OF THE SAT, ALL ANSWERS MUST BE IN THE LOWEST COMMON DENOMINATOR; THEREFORE, IF YOU ANSWER 4/6 AND EVEN IF THAT IS THE CORRECT ANSWER YOU WILL BE MARKED WRONG UNLESS YOU REDUCE AND RECORD YOUR ANSWER AS 2/3.]. THE NEGATIVE RECIPROCAL OF WHICH IS -2/1 WHICH EQUALS -2 WHICH IS THE SLOPE OF LINE g. NOW WE PLACE THE COORDINATE POINT (5,-7) OF LINE g ALONG WITH THE SLOPE OF LINE g -2 BACK INTO THE EQUATION OF A LINE FORMULA y = mx + b AND SOLVE FOR b. [REMEMBER: ONCE YOU HAVE THE EQUATION OF A LINE, ALL THE (x,y) COORDINATE POINTS ALONG THE LINE WILL FIT BACK INTO THE EQUATION OF THAT LINE.]. y = mx + b -7 = (5)(-2) + b -7 = -10 + b ADD 10 TO BOTH SIDES OF THE EQUATION, PRODUCING 3 = b.

EXAMPLE QUESTION: IN A LINEAR FUNCTION—LINEAR FUNCTION ALWAYS MEANS A LINE—LINE f PASSES THROUGH COORDINATE POINTS (-3,5) AND (2,-9). WHAT IS THE x INTERCEPT OF LINE f?

STEP ONE: ASCERTAIN THE SLOPE OF LINE f: -9 - 5/ 2 - (-3) [REMEMBER: WHENEVER YOU ARE SUBTRACTING A NEGATIVE NUMBER, SURROUND THAT NEGATIVE NUMBER IN PARENTHESES. THIS IS HOW YOU GET ALL THE 'GET-ABLE' QUESTIONS ON THE MATH PORTION OF THE SAT/ACT, AVOIDING CARELESS ERRORS.]. -9 - 5 = -14 BUT 2 - (-3) IS 2 + 3 WHICH EQUALS 5 THEREFORE THE SLOPE IS -14/5. TO ASCERTAIN THE y INTERCEPT CHOOSE (-3,5) OR (2,-9) AND PLUG BACK INTO THE EQUATION OF A LINE FORMULA, SOLVING FOR b: 5 = (-14/5)(-3) + b [RECALL: WHEN YOU MULTIPLY FRACTIONS, MULTIPLY THE NUMERATORS AND THE DENOMINATORS.]. 5 = 42/5 + b NOW MANIPULATE THE EQUATION TO SOLVE FOR b BY MOVING THE 42/5 FROM ONE SIDE OF THE EQUATION TO THE OTHER SIDE. BUT BEFORE WE CAN SUBTRACT 42/5 FROM 5 WE MUST TRANSFORM THE INTEGER 5 INTO A FRACTION. IN ORDER TO DO THIS FIRST WE NEED TO FIND A COMMON DENOMINATOR [RECALL: DIFFERENT RULES ALWAYS APPLY IN MATH TO ADDITION/SUBTRACTION versus MULTIPLICATION/DIVISION.]. [REMEMBER: IN ORDER TO TRANSFORM ANY INTEGER INTO A FRACTION SIMPLY PLACE A 1 IN THE DENOMINATOR.

HERE, THE INTEGER 5 TRANSFORMS INTO 5/1. NOW, MULTIPLY THE NUMERATOR IN 5/1 WITH THE DENOMINATOR IN 42/5 PRODUCING 25/5.

[EVEN WITH A MORE COMPLEX SITUATION, WE FOLLOW THE SAME RULES: 2/3 - 3/4 FIRST MULTIPLY THE DENOMINATORS OF THE TWO FRACTIONS PRODUCING 12 AS THE COMMON DENOMINATOR THEN TO TRANSFORM BOTH FRACTIONS INTO COMMON DENOMINATORS WE MULTIPLY THE NUMERATOR OF THE FIRST FRACTION WITH THE DENOMINATOR OF THE SECOND FRACTION PRODUCING 8 AND THE NUMERATOR OF THE SECOND FRACTION WITH THE DENOMINATOR OF THE FIRST FRACTION PRODUCING 9. NOW ADD 8/12 AND 9/12, REMEMBERING FOR ADDITION/SUBTRACTION OF FRACTIONS—UNLIKE RULES GOVERNING MULTIPLICATION/DIVISION OF FRACTIONS WHERE IN MULTIPLYING FRACTIONS WE SIMPLY MULTIPLY ACROSS: NUMERATOR TIMES NUMERATOR OVER DENOMINATOR TIMES DENOMINATOR; IN DIVISION OF FRACTIONS, WE MULTIPLY THE FIRST FRACTION BY THE RECIPROCAL OF THE SECOND FRACTION. FOR EXAMPLE, (8/3)(9/2) EQUALS 72/6 NOW DIVIDE NUMERATOR/DENOMINATOR BY 6 WHICH EQUALS 12/1 WHEREAS 8/3 / 4/6 WHICH EQUALS (8/3)(6/4) WHICH EQUALS 48/12 WHICH EQUALS 4—ONCE THERE IS A COMMON DENOMINATOR, WE SIMPLY

ADD OR SUBTRACT THE NUMERATORS PRODUCING 17/12.]

NOW SUBTRACT THE NUMERATORS FROM 42/5 MINUS 25/5 PRODUCING 42 - 25 = 17/5 WHICH IS THE y INTERCEPT OF THE LINE, BUT, WE NEED TO ASCERTAIN THE x INTERCEPT. TO DO THIS, WE NEED TO SUBSTITUTE 0 IN FOR y AND SOLVE FOR x.
$0 = (42/5)x + 17/5$.

AGAIN, UNDER RULES OF MANIPULATION OF COMPLEX ALGEBRAIC EQUATIONS,TO MOVE 17/5 FROM ONE SIDE OF THE EQUATION TO THE OTHER SIDE, SUBTRACT 17/5 FROM BOTH SIDES OF THE EQUATION LEAVING: $-17/5 = 42/5x$

[WHENEVER THE FRACTION OR INTEGER IS ATTACHED TO THE VARIABLE, IN ORDER TO MOVE THE FRACTION/INTEGER TO THE OTHER SIDE OF THE EQUATION, YOU MUST DIVIDE. HOWEVER, EASIEST MULTIPLICATION/DIVISION MANIPULATION RULE TO FOLLOW IS TO MULTIPLY BOTH SIDES OF THE EQUATION BY THE RECIPROCAL OF 42/5. (FOR THE PURISTS OUT THERE, I AM WELL AWARE OF THE FACT THAT WE ARE DIVIDING HERE. HOWEVER, WHY TAKE TWO STEPS WHEN YOU COULD MORE EASILY TAKE ONE STEP. IN OTHER WORDS, EVERY TIME YOU DIVIDE, YOUR FINAL STEP WILL ALWAYS BE TO MULTIPLY BOTH SIDES OF THE EQUATION BY THE

RECIPROCAL. {EVEN WHEN DIVIDING BY AN INTEGER, SAY 4/1, IN REALITY, YOU ARE MULTIPLYING BOTH SIDES OF THE EQUATION BY ITS RECIPROCAL: 1/4.}).

LEAVING: (5/42)(-17/5) = x   -17/42 = x

[WHEN YOU ARE MULTIPLYING TWO FRACTIONS WITH THE SAME (OR A MULTIPLE THEREOF) INTEGER IN THE NUMERATOR OF ONE FRACTION AND THE DENOMINATOR OF THE OTHER FRACTION, THESE TWO INTEGERS CANCEL EACH OTHER OUT. FOR EXAMPLE 2/3 TIMES 9/8 CROSS OUT THE 3 FROM THE DENOMINATOR OF THE FIRST FRACTION AND DIVIDE IT INTO THE 9 FROM THE NUMERATOR OF THE SECOND FRACTION PRODUCING 3 THEN CROSS OUT THE 2 FROM THE NUMERATOR OF THE FIRST FRACTION DIVIDE IT INTO THE 8 FROM THE DENOMINATOR OF THE SECOND FRACTION PRODUCING 4 RESULTING IN 3/4.] {OF COURSE YOU CAN ALWAYS MULTIPLY NUMERATORS/ DENOMINATORS OF THE TWO FRACTIONS PRODUCING 18/24 THEN REDUCE THIS FRACTION BY FINDING THE LARGEST COMMON FACTOR OF 18 AND 24 ‘6’ WHICH WHEN DIVIDED INTO 18 AND 24 PRODUCES 3/4.}

[PLEASE NOTE: ON THE SAT/ACT IT IS COMMON TO ANSWER MATH QUESTIONS REDUCING YOUR FRACTION ANSWER TO ITS LOWEST COMMON

DENOMINATOR. INDEED, ON THE SAT STUDENT-GENERATED QUESTIONS YOU MUST REDUCE YOUR ANSWER OR YOU WILL BE MARKED WRONG. FOR EXAMPLE, RECORDING 6/4 AS YOUR ANSWER ON AN SAT STUDENT-GENERATED QUESTION WILL RESULT IN AN UNFORCED ERROR. HERE, YOU MUST RECORD YOUR STUDENT-GENERATED ANSWER AS 3/2.]

THE FORMULA TO DETERMINE HOW MANY DEGREES ARE IN ANY POLYGON IS $180(n - 2)$ n STANDS FOR NUMBER OF SIDES.

IN A PARALLELOGRAM SAME SIDE ANGLES EQUAL 180 DEGREES AND THE ANGLES ACROSS FROM EACH OTHER ARE SAME DEGREE MEASURE.

WHEN YOU HAVE TWO PARALLEL LINES CUT BY A TRANSVERSAL, VERTICAL ANGLES ARE EQUAL AS ARE INTERIOR AND EXTERIOR CORRESPONDING ANGLES.

A LINE IS 180 DEGREES; A CIRCLE IS 360 DEGREES.

VOLUME/SURFACE AREA

VOLUME YOU ALWAYS MULTIPLY THREE QUANTITIES. SURFACE AREA IS THE AREA OF THE SURFACES. IF A CUBE HAS AN EDGE OF 4 [IN THREE DIMENSIONAL FIGURES WE SAY EDGE; IN TWO DIMENSIONAL FIGURES WE SAY SIDE.] WHAT ARE ITS VOLUME AND SURFACE AREA? VOLUME IS LENGTH TIMES WIDTH TIMES HEIGHT. IN A CUBE, THEY ARE ALL 4 THEREFORE VOLUME OF A CUBE IS 64. FOR SURFACE AREA OF A CUBE, THERE ARE SIX SIDES, AND THE AREA OF ALL THE SIDES IS THE SAME: AREA OF THE SQUARE WHICH IS SIDE SQUARED; THEREFORE, THE SURFACE AREA OF A CUBE IS 6s sq’d WHICH EQUALS 6(4 sq’d) = 6 (16) = 96

TRIANGLES

EQUILATERAL TRIANGLE MEANS EQUAL ANGLES 60 DEGREES AND EQUAL SIDE LENGTHS. ISOSCELES TRIANGLE HAS TWO SIDES OF EQUAL LENGTH AND THEREFORE TWO ANGLES OF EQUAL LENGTH. IN ANY TRIANGLE SMALLEST SIDE IS ACROSS FROM SMALLEST ANGLE, INTERMEDIATE SIDE ACROSS FROM INTERMEDIATE ANGLE AND LARGEST SIDE ACROSS FROM LARGEST ANGLE. WHENEVER YOU HAVE A SMALLER TRIANGLE INSIDE A LARGER TRIANGLE, THE SIDE LENGTHS OF THE TWO TRIANGLES ARE PROPORTIONAL.
FOR EXAMPLE, SMALLER TRIANGLE DBE IS INSIDE LARGER TRIANGLE ABC. IF SIDE AB IS 12 AND CORRESPONDING SIDE DB IS 7 AND SIDE AC IS 6 AND QUESTION ASKS FOR SIDE LENGTH OF CORRESPONDING SIDE DE ALWAYS SET UP A PROPORTION WITH LIKE ACROSS FROM LIKE.
$7/12 = x/6$ CROSS MULTIPLY TO SOLVE FOR x.

IN A TRIANGLE THE EXTERIOR ANGLE EQUALS THE SUM OF THE TWO INTERIOR ANGLES.

IN SIMILAR TRIANGLES, SIDE LENGTHS ARE PROPORTIONAL, ANGLE DEGREES ARE THE SAME.

BE AWARE THAT WHENEVER YOU SEE RADICAL 3 OR RADICAL 2 IN A TRIANGLE WE ARE DEALING WITH SPECIAL RIGHT TRIANGLES 30-60-90 OR 45-45-90.
IN A 30-60-90 TRIANGLE, THE SIDE OPPOSITE THE 30 DEGREE ANGLE IS x; THE SIDE OPPOSITE THE 60 DEGREE ANGLE IS x rad 3; AND THE SIDE OPPOSITE THE 90 DEGREE ANGLE IS 2x.

IN A 45-45-90 TRIANGLE, THE BOTH SIDES OPPOSITE THE 45 DEGREE ANGLE ARE s (SIDE LENGTH) AND THE SIDE OPPOSITE THE 90 DEGREE ANGLE IS s rad 2.

FOR ANY RIGHT TRIANGLE YOU CAN ALWAYS USE THE PYTHAGOREAN FORMULA: a sq'd + b sq'd = c sq'd WHERE a AND b ARE THE LEGS OF THE RIGHT TRIANGLE AND c IS THE HYPOTENUSE OF THE RIGHT TRIANGLE.

RECALL IN ANY RIGHT TRIANGLE 3-4-5 AND ALL MULTIPLES OF 3-4-5 ARE RIGHT TRIANGLES JUST AS 5-12-13 AND ALL MULTIPLES OF 5-12-13 ARE RIGHT TRIANGLES.

ARC LENGTH/SECTOR AREA

ARC LENGTH/SECTOR AREA FORMULA: BOTH ARC LENGTH AND SECTOR AREA INVOLVES EVALUATING A FRACTION OF A CIRCLE USING THE CENTRAL ANGLE. THE EASIEST WAY TO ENVISION THIS IS TO THINK ABOUT A 90 DEGREE ANGLE DRAWN FROM THE CENTER OF THE CIRCLE OUTWARD TO THE CIRCUMFERENCE OF THE CIRCLE. IN THIS INSTANCE, WE WOULD BE WORKING WITH A QUATER OF THE CIRCUMFERENCE (ARC LENGTH) OR A QUATER OF THE AREA OF THE CIRCLE (SECTOR AREA). THEREFORE, THE FORMULA IS AS FOLLOWS
CENTRAL ANGLE/360 TIMES CIRCUMFERENCE OF THE CIRCLE EQUALS ARC LENGTH AND TIMES AREA OF THE CIRCLE EQUALS SECTOR AREA.

SYSTEM OF EQUATIONS

WHENEVER YOU SEE TWO SYSTEMS OF EQUATIONS MANIPULATE, STACK AND SOLVE.

FOR EXAMPLE, $7x - 6y = -8$ $3x + 5y = 7$ IF THE QUESTION ASKS FOR THE VALUE OF x WE MUST ELIMINATE THE y AS FOLLOWS: PUT A PARENTHESES AROUND THE TOP EQUATION AND A 3 OUTSIDE THE PARENTHESES OF THE TOP EQUATION AND A PARENTHESES AROUND THE BOTTOM EQUATION AND A -7 OUTSIDE THE PARENTHESES OF THE BOTTOM EQUATION AND DISTRIBUTE (MULTIPLY NUMBER OUTSIDE PARENTHESES INSIDE THE PARENTHESES) AS FOLLOWS: $3(7x - 6y = -8)$ AND $-7(3x + 5y = 7)$ PRODUCING $21x - 18y = -24$ TOP $-21x - 35y = -49$ BOTTOM THEN ADD: $-53y = -73$ DIVIDE BOTH SIDES BY -53 [REMEMBER WHEN FRACTION OR INTEGER IS ATTACHED TO THE VARIABLE WITH NO PLUS OR MINUS SIGN AROUND YOU ALWAYS DIVIDE. THIS SITUATION IS MUCH DIFFERENT FROM
$3x - 7 - 6x = 2x - 4$ HERE WE ARE SURROUNDED BY PLUS AND MINUS SIGNS THEREFORE TO BEGIN OUR MANIPULATION OF ALGEBRAIC EQUATION, ADD 7 TO BOTH SIDES OF EQUATION PRODUCING
$3x - 6x = 2x + 3$ NOW COMBINE $3x$ WITH $-6x$ PRODUCING $-3x = 2x + 3$ THEN SUBTRACT $2x$ FROM

BOTH SIDES OF THE EQUATION PRODUCING -5x = 3. FINALLY, TO ISOLATE THE VARIABLE WE MUST DIVIDE BOTH SIDES OF THE EQUATION—WHATEVER YOU DO ON ONE SIDE OF THE EQUATION YOU MUST DO ON THE OTHER SIDE OF THE EQUATION—AND DIVIDE BOTH SIDES OF THE EQUATION BY -5 PRODUCING x = -3/5.

FOR EASIER SYSTEM OF EQUATION QUESTIONS, THE x OR y FROM EITHER OF THE TWO EQUATIONS IS A FACTOR OF THE x OR y FROM THE OTHER EQUATION.

FOR EXAMPLE, 8x - 3y = 11 2x + 5y = 6 WHAT IS THE VALUE OF y? WE NEED TO ELIMINATE THE x. NOTICE THAT 2 IS A FACTOR OF 8; THEREFORE, PLACE A -4 OUTSIDE PARENTHESES PLACED AROUND THE BOTTOM SYSTEM OF EQUATION PRODUCING -8x - 20y = -24 NOW ADD TOP/BOTTOM SYSTEM OF EQUATIONS PRODUCING -23y = -13 NOW DIVIDE BOTH SIDES BY -23 PRODUCING y = 13/23.

SYSTEM OF EQUATIONS: ONE SOLUTION v NO SOLUTION v INFINITE NUMBER OF SOLUTIONS

SYSTEMS OF EQUATIONS ARE TYPICALLY EQUATIONS OF LINES. IN THIS REGARD, WHEN THE QUESTION ASKS FOR A SOLUTION, THE SOLUTION MEANS WHERE THE TWO LINES INTERSECT. WHAT IS THE SOLUTION TO THE FOLLOWING SYSTEMS OF EQUATIONS: $-3x - 2y = 4$ AND $12x + 8y = 6$ HERE, WE CAN SEE THERE IS ONE SOLUTION MEANING THESE TWO LINES INTERSECT AT ONE POINT. FIRST, MANIPULATE THESE TWO EQUATIONS OF A LINE INTO $y = mx + b$ FORMAT—THE EQUATION OF A LINE—REMEMBERING THAT m IS THE SLOPE AND b IS THE y INTERCEPT—RECALL THAT THE y INTERCEPT IS WHERE THE x IS ZERO WHAT IS THE y COORDINATE VERSUS THE x INTERCEPT IS WHERE THE y IS ZERO WHAT IS THE x COORDINATE.

$-2x - 3y = 4$ AND $12x + 8y = 6$ FIRST, WE MUST MANIPULATE BOTH OF THESE EQUATIONS INTO $y = mx + b$ ADD 2x TO BOTH SIDES OF THE EQUATION PRODUCING $-3y = 2x + 4$ NOW DIVIDE BOTH SIDES OF THE EQUATION BY -3 PRODUCING $y = -2/3x - 4/3$ FOR OTHER EQUATION SUBTRACT 12x FROM BOTH SIDES OF THE EQUATION PRODUCING $8y = -12x + 6$ NOW DIVIDE BOTH SIDES BY 8 PRODUCING $y = -12/8x + 6/8$ [RECALL WE MUST REDUCE ALL FRACTIONS TO LOWEST COMMON

DENOMINATOR.] DIVIDE -12/8 BY 4 PRODUCING -3/2x AND DIVIDE 6/8 BY 2 PRODUCING 3/4. SINCE WE KNOW THESE TWO LINES INTERSECT AT ONE POINT THEY HAVE THE SAME x AND y COORDINATE POINT. THEREFORE, -2/3x - 4/3 = -3/2x + 3/4 NOW ADD 4/3 TO BOTH SIDES OF THE EQUATION, REMEMBERING ADDITION/SUBTRACTION OF FRACTIONS FIRST NEED COMMON DENOMINATOR—EASIEST WAY TO FIND COMMON DENOMINATOR IS TO MULTIPLY THE DENOMINATORS OF THE TWO FRACTIONS—3 TIMES 4 IS 12–NOW MULTIPLY THE NUMERATOR OF 3/4 WITH THE DENOMINATOR OF 4/3 PRODUCING 9/12 AND THE NUMERATOR OF 4/3 WITH THE DENOMINATOR OF 3/4 PRODUCING 16/12 NOW ONCE YOU HAVE THE SAME DENOMINATOR ADD THE NUMERATORS 9 +16 = 25 PRODUCING 25/12. NOW ADD -3/2x TO -2/3x—AGAIN WE NEED TO FIND A COMMON DENOMINATOR BY MULTIPLYING 2 TIMES 3 EQUALS 6. NOW MULTIPLY THE NUMERATOR OF -3/2 WITH THE DENOMINATOR OF -2/3 [RECALL WITH A NEGATIVE FRACTION YOU CAN CHOOSE TO MAKE THE NUMERATOR OR THE DENOMINATOR NEGATIVE BUT NOT BOTH.]. MULTIPLY -3 AND -3 PRODUCING 9 PLACE 9/6 THEN MULTIPLY THE NUMERATOR FROM 2/3 AND THE DENOMINATOR FROM 3/2 [REMEMBER WE ALREADY TOOK CARE OF THE NEGATIVES PRODUCING POSITIVE 9/6.]

WHERE THE QUESTION SAYS THERE IS NO SOLUTION, YOU ARE LOOKING FOR PARALLEL LINES [SAME SLOPE/DIFFERENT y INTERCEPT]

FOR EXAMPLE, $3x - 7y = -6$ and $5y - ax = 3$ HAS NO SOLUTION. WHAT IS THE VALUE OF a?

FIRST, MANIPULATE BOTH EQUATIONS ABOVE INTO EQUATION OF A LINE FORMAT. $3x - 7y = -6$ [SUBTRACT 3x FROM BOTH SIDES OF THE EQUATION] $-7y = -3x - 6$ [NOW DIVIDE BOTH SIDES BY -7] $y = 3/7x + 6/7$ NEXT MANIPULATE $5y - ax = 3$ [ADD ax TO BOTH SIDES OF THE EQUATION] $5y = ax + 3$ [NOW DIVIDE BOTH SIDES BY 5] $y = ax/5 + 3/5$. EVALUATING THE TWO EQUATIONS OF A LINE, WE HAVE DIFFERENT y INTERCEPTS, NEED THE SAME SLOPE. THEREFORE, SET $ax/5 = 3x/7$ AND SOLVE FOR x. [REMEMBER WHENEVER YOU HAVE TWO FRACTIONS FACING EACH OTHER ACROSS AN EQUALS SIGN, ALWAYS CROSS MULTIPLY.]. HERE $(ax)(7) = (5)(3x)$ $7ax = 15x$ [DIVIDE BOTH SIDES BY x] $7a = 15$ $a = 15/7$.

IF TWO LINES HAVE AN INFINITE NUMBER OF SOLUTIONS WE ARE LOOKING FOR THE SAME LINE. THEREFORE, WE NEED SAME SLOPE AND SAME y INTERCEPT.

PARABOLA

THE EQUATION OF A PARABOLA IS ALWAYS A QUADRATIC. IF THE PARABOLA IS POSITIVE OPENING UPWARD, THE VERTEX OF THE PARABOLA IS THE LOWEST COORDINATE POINT OF THE PARABOLA. IF THE PARABOLA IS NEGATIVE OPENING DOWNWARD, THE VERTEX OF THE PARABOLA IS THE HIGHEST POINT.

IN FINDING THE COORDINATE POINT OF THE VERTEX. ON THE SAT/ACT, THE EQUATION OF THE PARABOLA WILL EITHER BE GIVEN AS A TRADITIONAL QUADRATIC OR AS A MODIFIED QUADRATIC. FOR EXAMPLE, IF THE EQUATION IS PRESENTED AS: x sq'd — 5x — 24 FIRST, FACTOR THE QUADRATIC INTO (x — 8)(x + 3) NEXT DETERMINE THE ZEROS OR SOLUTIONS OF THE EQUATION WHICH ALWAYS MEANS THE VALUE(S) OF x THAT MAKE EACH PARENTHESES EQUAL ZERO. HERE THE SOLUTIONS ARE 8 AND -3. NEXT, AVERAGE THE TWO SOLUTIONS (8 + (-3)) / 2 WHICH EQUALS 5/2. THIS VALUE IS THE x COORDINATE OF THE VERTEX. NOW PLUG IN THE x COORDINATE BACK INTO THE EQUATION TO DETERMINE THE y COORDINATE OF THE VERTEX. (5/2 — 8)(5/2 + 3) WE NEED COMMON DENOMINATOR CONVERT 8 INTO 16/2 AND 3 INTO 6/2.

(5/2 — 16/2)(5/2 + 6/2) (-11/2)(11/2) = -121/4 WHICH IS THE y COORDINATE OF THE VERTEX.

THE NON-TRADITIONAL PARABOLA FORMULA IS AS FOLLOWS (x — 3) sq'd + 7 HERE WE KNOW 3 IS THE x COORDINATE POINT OF THE VERTEX BECAUSE
(x — 3)(x — 3) 3 PLUS 3 DIVIDED BY 2 IS 3. PLUGGING IN 3 WE ARE LEFT WITH 7 AS THE y COORDINATE OF THE VERTEX OF THE PARABOLA BECAUSE 0 + 7 = 7.

QUADRATICS

A QUADRATIC IS ONE TYPE OF POLYNOMIAL. QUADRATICS ARE ALWAYS EQUAL TO ZERO. THE SAT/ACT WILL REFER TO THE SOLUTION OF THE QUADRATIC—VALUE OF THE x's THAT MAKE EACH PARENTHESES EQUAL ZERO—AS ROOTS, ZEROS OR SOLUTIONS. ON THE SAT/ACT QUADRATICS ARE TESTED IN THREE WAYS: FACTORING, QUADRATIC EQUATION, DIVISION OF POLYNOMIALS.

THE MOST COMMON METHOD ON THE SAT/ACT TO SOLVE QUADRATICS IS TO FACTOR THE QUADRATIC.

x sq'd — 8x + 12

STEP ONE: PLACE TWO SETS OF PARENTHESES FOCUSING ON FACTORING x sq'd INTO (x   )(x   )

STEP TWO: GO TO THE PLUS/MINUS SYMBOL BEFORE THE LAST INTEGER, IF THE SYMBOL IS MINUS THEN ONE PARENTHESES IS PLUS, ONE IS MINUS. IF THE SYMBOL IS PLUS LOOK TO THE SYMBOL BEFORE THE SECOND INTEGER, IF THAT SYMBOL IS PLUS, BOTH PARENTHESES ARE PLUS, IF THAT SYMBOL IS MINUS BOTH PARENTHESES ARE MINUS.

STEP THREE: HERE WE PLACE TWO MINUS SIGNS BECAUSE THE SYMBOL BEFORE THE LAST INTEGER IS PLUS BUT BEFORE THE SECOND INTEGER IS MINUS.

STEP FOUR: GO THE LAST INTEGER, HERE IT IS 12 AND THINK ABOUT THE FACTORS OF 12 THAT ADD UP TO 8: 6 AND 2.

FINAL STEP: SOLVE: (x — 6)(x — 2)

SINCE ALL QUADRATICS ARE EQUAL TO 0, WHEN THE QUESTION ASKS FOR THE SUM OR PRODUCT OR DIFFERENCE OF THE SOLUTIONS OR THE ZEROS OF THE QUADRATIC EQUATION, YOU MUST PUT IN THE VALUE WHICH WILL RESULT IN BOTH SET OF PARENTHESES EQUAL TO ZERO. HERE 6 AND 2.

WHENEVER YOU SEE x sq'd MANIPULATE THE PROBLEM SO THAT IT EQUALS ZERO. FOR EXAMPLE, x sq'd + 3x = 40 MANIPULATE INTO QUADRATIC EQUATION BY SUBTRACTING 40 ON BOTH SIDES OF THE EQUATION. NOW WE HAVE x sq'd + 3x — 40 = 0. FACTORING THIS QUADRATIC WE BEGIN BY SETTING UP TWO SETS OF PARENTHESES (x)(x). NOW, GO TO THE SYMBOL BEFORE THE LAST INTEGER WHICH IS MINUS; THEREFORE, WE RECALL ONE PARENTHESES IS PLUS, THE OTHER IS MINUS. NEXT, THINK OF THE FACTORS OF 40 THAT DIFFER BY

THREE: 8 AND 5. BECAUSE THE SECOND INTEGER IS POSITIVE 3 WE HAVE (x + 8)(x — 5). FINALLY, YOU CAN ALWAYS CHECK YOUR WORK BY F.O.I.L.'ING YOUR ANSWER: MULTIPLYING THE FIRST, OUTER, INNER, LAST INTEGER/VARIABLE TO REPLICATE THE PROBLEM. HERE (x)(x) = x sq'd (x)(-5) = -5x (8)(x) = 8x AND (8)(-5) = -40. PUTTING IT TOGETHER WE HAVE x sq'd + 3x — 40.

SIMPLIFY AND/OR FACTOR QUADRATIC EQUATION:

(3x — 9) / (x sq'd — 9)

STEP ONE: FACTOR 3 OUT OF (3x — 9) TO GET 3(x — 3)

STEP TWO: FACTOR (x sq'd — 9) INTO (x — 3)(x + 3)

[PLEASE NOTE: WHENEVER YOU HAVE TWO PERFECT SQUARES JOINED BY A MINUS SIGN, FOR EXAMPLE (x sq'd — 25) YOU WILL ALWAYS FACTOR INTO: (x — 5)(x + 5).]

WE NOW HAVE: 3(x — 3) / (x — 3)(x + 3) CROSSING OUT (x — 3) FROM THE NUMERATOR AND THE DENOMINATOR LEAVES 3 / (x + 3) AS THE ANSWER.

NOT ALL QUADRATICS WILL FACTOR. SOMETIMES YOU WILL NEED TO EMPLOY THE QUADRATIC EQUATION. HOW WILL YOU KNOW? LOOK FOR A RADICAL IN THE ANSWER CHOICES.

QUADRATIC EQUATION

-b (plus/minus) sq root b sq'd — 4ac / 2a

x sq'd — 8x + 3

THE VALUE ATTACHED TO THE x sq'd IS ALWAYS a; THE VALUE ATTACHED TO x IS ALWAYS b; AND, THE LAST VALUE IS ALWAYS c.

BEST APPROACH IS TO WRITE DOWN QUADRATIC EQUATION AND TO LIST THE VALUES OF a, b, AND c. HERE, a EQUALS 1, b EQUALS -5, AND c EQUALS 3.

(-(-8)) / (2)(1) EQUALS 8/2 EQUALS 4

[I ALWAYS RECOMMEND PLACING THE 2a UNDERNEATH BOTH PARTS OF THE EQUATION AS A QUICK WAY TO NARROW ANSWER TO TWO CHOICES.]

sq root b sq'd — 4ac / 2a
EQUALS sq root (-8) sq'd — (4)(1)(3) / (2)(1)
EQUALS sq root 64 —12 / 2
EQUALS sq root 52 / 2
[IN ORDER TO SIMPLIFY YOU MUST FIND LARGEST PERFECT SQUARE THAT DIVIDES INTO 52 — HERE 4 — REWRITE AS sq root (4)(13) / 2 SINCE THE SQUARE ROOT OF 4 IS 2 WE HAVE 2 sq root 13 / 2 CROSS OUT THE 2 FOR sq root 13. ANSWER IS 4 + sq root 13 AND 4 — sq root 13.

DISCRIMINANT

THE OTHER METHOD OF DETERMINING THE NUMBER OF DISTINCT SOLUTIONS TO A QUADRATIC EQUATION IS TO USE THE DISCRIMINANT: b sq'd — 4ac

IF THE ANSWER IS POSITIVE THERE ARE TWO DISTINCT SOLUTIONS; IF THE ANSWER IS ZERO THERE IS ONE SOLUTION; IF THE ANSWER IS NEGATIVE THERE IS NO SOLUTION.

IN THE QUESTION ABOVE, (-8) sq'd — (4)(1)(3) EQUALS 64 — 12 = 52 THEREFORE WE KNOW THERE ARE TWO DISTINCT SOLUTIONS.

FOR EXAMPLE, x sq'd + 4x + 4 PLUGGING INTO DISCRIMINANT EQUATION PRODUCES
(4) sq'd — (4)(1)(4)   16 — 16 = 0 THEREFORE, THERE IS ONE SOLUTION.

HOWEVER, x sq'd — 5x + 7 PRODUCES
(-5) sq'd — (4)(1)(7) = 25 — 28 = -3 THEREFORE, THERE IS NO SOLUTION

DIVISION OF POLYNOMIALS

WHEN YOU SEE (x sq'd — 3x + 6) / (x — 4) FIRST LOOK AT THE ANSWER CHOICES: WHEN ANSWER CHOICE LOOKS LIKE x + 1 + 10 / (x — 4) YOU KNOW WE NEED TO DIVIDE POLYNOMIALS.

STEP ONE: JUST LIKE LONG DIVISION, WE ARE DIVIDING x sq'd — 3x + 6 BY x — 4 COMPARE x WITH x sq'd IN ORDER TO GO FROM x TO x sq'd WE MUST MULTIPLY (x — 4)(x) PRODUCING x sq'd — 4x. NEXT, PLACE x sq'd — 4x UNDERNEATH x sq'd — 3x + 6 AND SUBTRACT (x sq'd — 3x + 6) — (x sq'd — 4x) PRODUCES -3x + 4x WHICH EQUALS x + 6. PLACE x AS PART OF YOUR ANSWER. NOW MULTIPLY (x — 4) BY 1 PRODUCING (x — 4) PLACE UNDERNEATH (x + 6) AND SUBTRACT PRODUCING A REMAINDER OF 10 AND KEEPING x ADDING + 1 AS PART OF THE ANSWER. FINALLY PLACE 10 / (x — 4) AS REMAINDER PRODUCING AN ANSWER OF x + 1 + 10/(x — 4).

ALTERNATIVE METHOD: F.O.I.L. (x — 4)(x + 1) PRODUCING x sq'd + x — 4x — 4 NOW ADD REMAINDER OF 10 WHICH EQUALS x sq'd — 3x + 6.

IRRATIONAL NUMBERS

TREAT i JUST LIKE ANY OTHER VARIABLE WHEN ADDING OR SUBTRACTING. FOR EXAMPLE, 7i — 4i = 3i. IMPORTANT TO KNOW THAT i sq'd EQUALS -1.

(3i — 5) / (6 — 4i) YOU MUST MULTIPLY THE NUMERATOR AND DENOMINATOR BY THE CONJUGATE: 6 + 4i PRODUCING ((3i — 5) / (6 — 4i))((6 + 4i) / (6 + 4i)) USING THE F.O.I.L. METHOD, (3i — 5)(6 + 4i) = 18i + 12i sq'd — 30 — 20i [REPLACE i sq'd WITH -1] COMBINE LIKE TERMS -2i — 42 IN THE NUMERATOR AND (6 — 4i)(6 + 4i) EQUALS 36 + 24i — 24i — 16i sq'd EQUALS 36 — (16)(-1) EQUALS 36 + 16 = 52 THEREFORE WE HAVE (-2i — 42) / 52

COMPLEX FRACTIONS

RECALL THAT 2/3 — 3/4 REQUIRES US TO GET A COMMON DENOMINATOR BY MULTIPLYING (3)(4) = 12 TO FIND THE NUMERATORS SIMPLY MULTIPLY THE DENOMINATOR OF THE FIRST FRACTION (3) WITH THE NUMERATOR OF THE SECOND FRACTION (3) AND THE DENOMINATOR OF THE SECOND FRACTION (4) WITH THE NUMERATOR OF THE FIRST FRACTION (2) PRODUCING 9 + 8 / 12 = 17/12.

MATH PRINCIPLES ARE ETERNAL, MEANING NO MATTER HOW COMPLEX THE QUESTION, WE ALWAYS FOLLOW THE SAME RULES.

(3 / (x — 2)) — (4 / 5)

STEP ONE: COMMON DENOMINATOR MULTIPLY (x — 2)(5) [PLEASE NOTE: WHEN FRACTION IS NEGATIVE YOU CAN CHOOSE TO MAKE EITHER THE NUMERATOR OR THE DENOMINATOR NEGATIVE.] HERE WE HAVE 5x — 10 AS THE COMMON DENOMINATOR. NOW MULTIPLY (x — 2)(-4) AND (3)(5) PRODUCING -4x + 8 AND 15 WHICH EQUALS (-4x + 23) / (5x — 10).

SOMETIMES YOU WILL HAVE A QUADRATIC IN THE NUMERATOR OR THE DENOMINATOR. TRY TO FACTOR OR PLUG IN ANSWER CHOICES TO SOLVE.

EXPONENTS

STEP ONE YOU NEED THE SAME BASE. IF QUESTION SAYS (9)(3) (12TH POWER) CHANGE 9 TO 3 sq'd THEN MULTIPLY BY ADDING 2 PLUS 12 EQUALS
3 (14TH POWER)

WHEN YOU MULTIPLY EXPONENTS WITH SAME BASE, ADD THE EXPONENTS.

WHEN YOU DIVIDE EXPONENTS WITH SAME BASE, SUBTRACT THE EXPONENTS.

WHEN YOU SQUARE, CUBE, ETC EXPONENTS WITH SAME BASE MULTIPLY THE EXPONENTS.

WHEN YOU TAKE SQUARE ROOT, CUBE ROOT, ETC EXPONENTS WITH SAME BASE DIVIDE THE EXPONENTS.

FRACTION EXPONENTS THE POWER IS THE NUMERATOR AND THE ROOT IS THE DENOMINATOR. USE RULES ABOVE TO SOLVE (x (2/3 POWER)) / (x (3/4 POWER)) WHEN YOU HAVE THE SAME BASE WORK WITH THE EXPONENTS—HERE 2/3 MINUS 3/4—GET COMMON DENOMINATOR OF 12 SINCE (2)(4) IS 8 AND (3)(3) IS 9 WE HAVE 8/12 MINUS 9/12 EQUALS -1/12.

NEGATIVE EXPONENTS REQUIRE YOU TO USE THE RECIPROCAL. HERE SINCE WE HAVE 1/x (-1/12 POWER) THE NEGATIVE EXPONENT TURNS SOLUTION INTO x (1/12 POWER).

F.O.I.L.'ING COMPLEX POLYNOMIALS

(x — 3)(x + 5)(x — 7)

STEP ONE: F.O.I.L. TWO OF THE THREE PARENTHESES: x sq'd + 5x — 3x —15 EQUALS (x sq'd + 2x — 15) NOW MULTIPLY THIS TIMES (x — 7) THROUGH DISTRIBUTION: (x sq'd)(x) + (2x)(x) — (15)(x) EQUALS
(x cubed + 2x sq'd — 15x)(-7) EQUALS
(-7x cubed — 14x sq'd + 105x)

RATIOS

JJ HAS COLORED MARBLES IN THE RATIO OF 5 BLUE TO 7 GREEN TO 2 RED. IF JJ HAS 98 MARBLES, HOW MANY ARE GREEN. ALWAYS SET UP RATIO AS PART/WHOLE TO PART/WHOLE OR PART/PART TO PART/PART DEPENDING ON THE QUESTION. HERE THE RATIO QUESTION IS PART WHOLE. ALSO, WE SET UP RATIO ON ONE SIDE SUBSTANCE ON OTHER SIDE.
HERE 7/14 OR 1/2 = x/98 REMEMBER TO CROSS MULTIPLY PRODUCING 2x = 98 DIVIDE BY 2 x = 49.
IF CHIL HAD 30 RED MARBLES HOW MANY BLUE MARBLES DID DIL HAVE. THIS IS PART TO PART: 2/5 = 30/x CROSS MULTIPLY (5)(30) = (2)(x) EQUALS 150/2 = 75.

FINALLY, IF QUESTION SAID SAL HAS FISH TO TURTLES IN THE RATIO OF 2:5 AND TURLES TO BIRDS IN THE RATIO OF 15:9. WHAT IS THE RATIO OF FISH TO BIRDS. WE NEED TO HAVE SAME RATIO OF TURTLES; THEREFORE, MULTIPLY 2 AND 5 BY 3 PRODUCING 6:15 THEREFORE THE RATIO OF FISH TO BIRDS IS 6:9 OR 2:3.

PROBABILITY

PROBABILITY IS ALWAYS WHAT YOU ARE LOOKING FOR / ALL POSSIBLE

ABSOLUTE VALUE

ALWAYS SET ANSWER EQUAL TO THE POSITIVE AND NEGATIVE SOLUTIONS. WHAT IS THE SUM OF THE SOLUTIONS TO $|3x - 7| = 2$?

Solve $3x - 7 = 2$ [ADD 7 TO BOTH SIDES OF THE EQUATION] $3x = 9$ [NOW DIVIDE BOTH SIDES BY 3] $x = 3$ AND $3x - 7 = -2$ [ADD 7 TO BOTH SIDES OF THE EQUATION] $3x = 5$ [DIVIDE BOTH SIDES BY 5] $x = 5/3$ FINALLY, ADD 3 PLUS 5/3 CONVERTING 3 TO 9/3 PLUS 5/3 EQUALS 14/3.

LESS THAN/GREATER THAN

$3x - 7 < x + 2 < 2x - 5$

TREAT LESS THAN AND GREATER THAN SYMBOLS AS EQUALS SIGN AND SOLVE. ONCE YOU SOLVE BRING LESS THAN/GREATER THAN BACK INTO THE EQUATION.

$3x - 7 = x + 2$ [ADD 7 TO BOTH SIDES AND SUBTRACT x FROM BOTH SIDES] $2x = 9$ $x = 9/2$

$x + 2 = 2x - 5$ [ADD 5 TO BOTH SIDES OF EQUATION AND SUBTRACT x FROM BOTH SIDES OF THE EQUATION] $x = 7$

NOW WE KNOW BRINGING LESS THAN AND GREATER THAN BACK INTO THE EQUATION $9/2 < x < 7$

MEAN, MEDIAN, MODE, STANDARD DEVIATION

MODE IS MOST COMMON VALUE IN GROUP OF VALUES.

MEDIAN IS ROUGHLY ‘MIDDLE NUMBER’. TO DETERMINE THE MEDIAN:

STEP ONE EVEN OR ODD NUMBER OF VALUES. WITH ODD NUMBER OF VALUES, DIVIDE BY 2 AND ROUND UP. THEREFORE, 9 VALUES, MEDIAN IS 4.5 OR FIFTH VALUE. WITH EVEN NUMBER OF VALUES AVERAGE THE TWO VALUES IN THE MIDDLE AS FOLLOWS: WITH 10 VALUES DIVIDE 10 BY 2 EQUALS 5, AVERAGE FIFTH AND SIXTH VALUES. [BE CAREFUL OF FREQUENCY WHICH MUST BE TAKEN INTO ACCOUNT IN DETERMINING MEDIAN.]. ONCE YOU DETERMINE WHICH VALUE YOU ARE LOOKING FOR, LINE UP VALUES FROM LOWEST, ONCE YOU HIT THAT VALUE—THE FIFTH WITH NINE VALUES—THAT VALUE IS THE MEDIAN.

MEAN (AVERAGE) IS SUM OF THE NUMBERS OVER NUMBER OF NUMBERS

STANDARD DEVIATION—THE MORE THE NUMBERS ARE SPREAD OUT THE HIGHER THE STANDARD DEVIATION; CONVERSELY, THE MORE THE NUMBERS ARE BUNCHED UP IN THE MIDDLE, THE LOWER THE STANDARD DEVIATION.

DISTANCE PYRAMID

DISTANCE EQUALS RATE TIMES TIME. RATE EQUALS DISTANCE/TIME AND TIME EQUALS DISTANCE/RATE.

GRAPHS

WHEN YOU ARE PROVIDED A GRAPH OF AN EQUATION OF A LINE OR A PARABOLA, PLUG IN DEFINITE COORDINATE POINTS OF THE LINE OR PARABOLA. IF THERE ARE NO DEFINITE POINTS PLUG IN APPROXIMATE COORDINATE POINTS.

WHERE QUESTION ASKS $f(x) = 5$ [RECALL WHATEVER LETTER—x, h, t, etc.—IS IN THE PARENTHESES IS THE x COORDINATE AND WHAT IT EQUALS IS THE y COORDINATE] QUESTION IS ASKING FOR ALL VALUES OF x ON THE GRAPH WHERE THE y VALUE IS 5. CONVERSELY, WHERE QUESTION ASKS f(5) EQUALS

GO TO GRAPH FIND WHERE x COORDINATE IS 5 AND PROVIDE MATCHING y COORDINATE AS YOUR ANSWER.

IDENTIFYING BOUNDARIES OF SHADED AREA CONTAINED ON A GRAPH

SELECT FOUR DEFINITE COORDINATE POINTS CONTAINED WITHIN THE BOUNDARIES OF THE SHADED REGION—NOT ALONG THE LINES CREATING SHADED REGION, BUT FOUR DEFINITE COORDINATE POINTS WITHIN THE SHADED REGION.

THEN, SIMPLY PLUG IN ALL FOUR COORDINATE POINTS INTO THE LESS THAN/GREATER THAN EQUATIONS ENSURING THAT ALL FOUR POINTS FIT WITHIN THE BOUNDARIES OF THE LESS THAN/ GREATER THAN EQUATIONS ATOP THE QUESTION.

TRANSFORMATION OF A GRAPH

WHEN YOU ARE GIVEN THE EQUATION OF A LINE OR A PARABOLA ADDITION INSIDE THE PARENTHESES

SHIFTS THE GRAPH TO LEFT, SUBTRACTION INSIDE PARENTHESES SHIFTS THE GRAPH TO THE RIGHT [THINK OPPOSITE OF HOW YOU IMAGINE IT WOULD BE]. OUTSIDE THE PARENTHESES OF EQUATION OF LINE OR PARABOLA ADDITION SHIFTS GRAPH UPWARD, SUBTRACTION SHIFTS GRAPH DOWNWARD.

TRIGONOMETRY

SOH-CAH-TOA

SINE—OPPOSITE/HYPOTENUSE
COSINE—ADJACENT/HYPOTENUSE
TANGENT—OPPOSITE/ADJACENT

IN TRIGONOMETRY, QUADRANT I GOES FROM 0 DEGREES TO 90 DEGREES, MAKING 90 DEGREES 1/2 PI. QUADRANT II GOES FROM 90 TO 180 DEGREES, MAKING 180 DEGREES PI.

SINE 0 DEGREES IS 0, 30 DEGREES IS 1/2,
45 DEGREES IS 1/rad 2, 60 DEGREES IS rad 3/2,
90 DEGREES IS 1, 120 DEGREES IS rad 3/2, 135 DEGREES IS 1/rad 2, 150 DEGREES IS 1/2, AND 180 DEGREES IS 0.

COSINE 0 DEGREES IS 1, 30 DEGREES IS rad 3/2,
45 DEGREES IS 1/rad 2, 60 DEGREES IS 1/2, 90 DEGREES IS 0, 120 DEGREES IS -1/2, 135 DEGREES IS -1/rad 2, 150 DEGREES IS -rad3/2, AND 180 DEGREES IS -1.

TANGENT 0 DEGREES IS 0, 30 DEGREES IS 1/rad 3,
45 DEGREES IS 1, 60 DEGREES IS rad 3, 90 DEGREES IS NOT DEFINED, 120 DEGREES IS -rad 3, 135 DEGREES IS -1, 150 DEGREES IS -1/rad 3, AND 180 DEGREES IS 0.